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PRIMITIVITY AND ENDS OF GRAPHS

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A graph X is said to be *primitive* if its automorphism group G acts primitively on the vertex set VX; that is, the only G-invariant equivalence relations on VX are the one where all the classes have size one and the equivalence relation which has only one class, the whole of VX. We investigate the end structure of locally finite primitive graphs. Our main result shows that it has a very simple description; in particular, locally finite primitive graphs are accessible in the sense of Thomassen and Woess.

Introduction

A graph is said to be *transitive* (or *vertex-transitive*) if the automorphism group acts transitively on the vertex set. In general it looks like transitive graphs with more than one end are well behaved and gentle things. But, things are not always as they seem: a recent result of Dunwoody [3] shows that their structure can be very complicated.

Following [9] we say that a locally finite graph Γ is *accessible* if there is a number k such that any two ends of Γ can be separated by removing k, or fewer, edges from Γ. Here we can of course replace "edges" with "vertices" and we will get the same concept. The well known accessibility conjecture of Wall, can now be rephrased as the conjecture that the Cayley-graphs (with respect to some finite generating set) of any finitely generated group are accessible. This was an open problem for twenty years and generally believed to be true, until Dunwoody [3] gave a counterexample.

So, mere transitivity is not enough to guarantee good behaviour. On the other hand, there are various results (see e.g. [9]) showing that if we add more assumptions about the action of the automorphism group the situation becomes easier. Often we are even able to get explicit descriptions of the possible graphs.

In this paper we study locally finite graphs where the automorphism group acts primitively on the vertex set. Such graphs have previously been studied by Jung and Watkins in [4] and [5]. The local structure can be very complicated, but we show that the end structure is simple (Theorem 4). In particular these graphs must be accessible (Theorem 3).

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In the first section we introduce notation and review briefly the basic concepts and results used in this paper. Next we relate the theory of structure trees to primitivity, and then, in the third and final section, prove our main results.

1. Preliminaries

In this paper the term *graph* denotes an undirected graph without multiple edges or loops, so we can think of a graph Γ as a pair $(V\Gamma, E\Gamma)$, where $V\Gamma$ is the set of vertices and $E\Gamma$ is a set of two element subsets of $V\Gamma$. Let $d_\Gamma(-,-)$ denote the usual distance on $V\Gamma$ and let $\deg_\Gamma(-)$ denote the degree of a vertex in Γ. The connectivity of a graph Γ, denoted by $\kappa(\Gamma)$, is defined as the minimum number of vertices we need to remove from Γ to get a disconnected graph with the exception that $\kappa(K_n) := n-1$. A *block* of Γ is a maximal subgraph with connectivity higher than 1, or an edge e together with its two endpoints if the edge e belongs to no subgraph of Γ with connectivity higher than 1. It is easy to prove that two edges belong to the same block if and only if they belong to a common non-trivial cycle.

1.1 Ends

There are several different ways of defining the ends of a graph (see e.g. [6]). The most simple way to define the ends of a graph Γ is to define them as equivalence classes of half-lines (a *half-line* is a family $\{\alpha_i\}_{i\in\mathbf{N}}$ of distinct vertices such that α_i is adjacent to α_{i+1} for all $i \in \mathbf{N}$). We say that two half-lines L_1 and L_2 are in the same end if there are infinitely many disjoint paths connecting vertices in L_1 to vertices in L_2. This is easily proved to be an equivalence relation. The equivalence classes are the *ends* of our graph. We can also notice that two half-lines L_1 and L_2 are not in the same end if and only if we can find a finite set F of vertices (or edges) and distinct components C_1 and C_2 of $\Gamma \setminus F$ such that C_1 contains all but finitely many of the vertices in L_1 and C_2 contains all but finitely many of the vertices in L_2. With a little thought one sees that every half-line in the same end as L_1 must also have all but finitely many of its vertices contained in C_1. So we can say that the end that L_1 belongs to is contained in C_1, and that the set F *separates* the ends that L_1 and L_2 belong to. Denote the set of ends that is contained in C_1 with ΩC_1. The maximum number of disjoint half-lines contained in an end ω is called the *thickness* of the end ω. If this number is finite then we say that the end is *thin*, otherwise, if this number is infinite, we say that the end is *thick*. The end of the graph $\mathbf{Z}\times\mathbf{Z}$ is thick but the ends of $\mathbf{Z}$ are both thin. Note that a graph has more than one end if and only if there is a finite set F of vertices such that $\Gamma \setminus F$ has more than one infinite component.

There is also a different way to view the ends of a graph. Then we think of the ends as a boundary; in the locally finite case we can take the ends as the ideal points of a certain compactification of $V\Gamma$ with the discrete topology. In particular we get a natural topology on the set of ends: a subbasis for this topology is the set of all sets ΩC where C is a component of $\Gamma \setminus F$ for some finite set F of vertices.

1.2 Structure trees

The questions that we are concerned with in this paper are only meaningful for graphs with more than one end. The most powerful tool available to study the structure of such graphs and their automorphism groups is the theory of "structure trees". The basic construction is described in the book by Dicks and Dunwoody [1, Chapter II]. How properties of the graph and the automorphism group relate to a structure tree is further described in [6] and [9]. Our notation will follow that of [1] and [6]. Luckily enough we do not need the full force of the results proved in [1], and can employ a simpler setup.

Let Γ be a graph. For a subset $s \subseteq V\Gamma$ we set $s^* := V\Gamma \setminus s$. The *co-boundary* δs of s is defined as the set of edges from s to s^*. A subset $s \subseteq V\Gamma$ such that $|\delta s| < \infty$ is called a *cut*.

Theorem 1. [2, Theorem 1.1] *Let Γ be a locally finite graph with more than one end. Then there exists a cut e such that both e and e^* are connected, and such that for every $g \in \mathrm{Aut}(\Gamma)$ one of the sets*

$$e \cap eg, \; e \cap e^*g, \; e^* \cap eg, \; e^* \cap e^*g$$

is empty.

A cut satisfying the conditions in the theorem above is called a D-*cut*. If e is a D-*cut* then the set

$$E := \{e\mathrm{Aut}(\Gamma), e^*\mathrm{Aut}(\Gamma)\}$$

is an example of what is called a *tree set*.

When we talk about trees it is convenient to think of an edge $\{\alpha, \beta\}$ as a pair of directed edges $e = (\alpha, \beta)$ and $e^* = (\beta, \alpha)$. We will let ET denote the set of directed edges in T. If $e = (\alpha, \beta)$ is an edge in T then define the *origin* $o(e)$ as the vertex α and the *terminus* $t(e)$ as β. The reason for the name *tree set* is that it is possible to construct a tree $T = T(E)$ such that T bears a close resemblance to Γ and such that we can identify E and ET. The details of the construction of T can be found in [1] and also in [9]. Here we will just briefly describe the properties of T that we will be using.

We will not be distinguishing between elements of E and elements of ET. The first thing we insist on is that if e is identified with an edge (α, β) then e^* is identified with (β, α). The basic idea behind the construction of T is that if $e, f \in E$ and $f \neq e^*$ then $t(e) = o(f)$ if and only if $f \subset e$ and there is no g in E such that $f \subset g \subset e$. From this it is clear that the action of $\mathrm{Aut}(\Gamma)$ on E gives us an action of $\mathrm{Aut}(\Gamma)$, as a group of automorphisms, on T. Now we can define a map $\phi : V\Gamma \to VT$, which will commute with the action of $\mathrm{Aut}(\Gamma)$. The idea is simple: we find a minimal element e of E subject to containing a vertex α, and then we set $\phi(\alpha) := t(e)$. It is easily proved that $\phi(\alpha)$ does not depend on the choice of e and that any edge in T that contains α will point towards α. Two vertices α and β in Γ are mapped by ϕ to different vertices in T if and only if there is an element $e \in E$ such that $\alpha \in e$ but $\beta \notin e$.

1.3 Primitivity

Our group theoretic notation is fairly standard. Let a group G act on a set Ω. We write the group action on the left and use G_α to denote the *stabilizer* of an element $\alpha \in \Omega$ in G. If $\Delta \subseteq \Omega$ then then the *setwise stabilizer* of Δ, denoted by $G_{\{\Delta\}}$, is defined as the subgroup of G consisting of all $g \in G$ such that $\Delta g = \Delta$. The group G is said to act *primitively* on Ω if G acts transitively and the only G-invariant equivalence relations on Ω are the trivial one, where all classes have size one, and the one which has only one class Ω. The following is very well known.

Proposition 1. *Let G be a group acting transitively on a set Ω. Then the following are equivalent:*
(i) *G acts primitively on Ω;*
(ii) *for all $\alpha \in \Omega$ the stabilizer G_α is a maximal proper subgroup of G;*
(iii) *for any pair $\alpha, \beta \in \Omega$ the graph $(\Omega, \{\alpha, \beta\}G)$ is connected.*

If Γ is a graph then we say that Γ is primitive if $G := \mathrm{Aut}(\Gamma)$ acts primitively on $V\Gamma$. All null graphs (graphs with no edges) are primitive, but we can see from (iii) that all other primitive graphs are connected. Condition (iii) above will be very useful: it allows us to choose a different edge set without loosing connectivity. Thus we are able to bring to the surface properties that might have been hiding behind overabundance of edges. In particular if α, β are distinct vertices of Γ then $(V\Gamma, \{\alpha, \beta\}G)$ is then quasi-isometric to Γ and therefore has the same end structure (cf. [7]). (Two graphs Γ, Γ' are *quasi-isometric* (some authors use *roughly-isometric*) if there exists a map $\phi: V\Gamma \to V\Gamma'$ and constants a, $b > 0$ such that for all $\alpha, \beta \in V\Gamma$ we have $\frac{1}{a} d_\Gamma(\alpha, \beta) - b \le d_{\Gamma'}(\phi(\alpha), \phi(\beta)) \le a \cdot d_\Gamma(\alpha, \beta) + b$.)

2. Structure trees and primitivity

From now on Γ is a locally finite primitive graph with infinitely many ends. Put $G := \mathrm{Aut}(\Gamma)$. Let e_0 be D-cut and $E := e_0 G \cup e_0^* G$. Furthermore, we let $T := T(E)$ be the structure tree and let

$$\phi : V\Gamma \to VT;$$

denote the associated map (see above).

Since ϕ is G-equivariant, the fibres of ϕ will define a G-invariant equivalence relation on $V\Gamma$. It is clearly impossible that ϕ maps all vertices of Γ to the same vertex of T. Now the primitivity of Γ implies that each fibre of ϕ contains only one vertex, and therefore

ϕ is injective.

Thus we can (and will) identify vertices in Γ with their ϕ images in T.

From the way E is defined it is apparent that G has at most two orbits on E. Thus G has also at most two orbits on the (directed) edges of T, and therefore at most two orbits on the vertices of T. If G acted transitively on VT then $\mathrm{Im}\,\phi$ would be equal to VT, because $\mathrm{Im}\,\phi$ is G-invariant. But, then the parts of the natural bipartition of T would define a non-trivial G-invariant equivalence relation on $V\Gamma$. Thus we have

$$\text{Im}\,\phi \textit{ is equal to one of the parts of the natural bipartition of } T.$$

Now we see that G acts on T without inversion (transposes no pair of adjacent vertices) and with fundamental domain a graph with only two vertices and two directed edges between them, one going each way. Then the Bass-Serre theory of groups acting on trees tells us that if $\alpha \in V\Gamma$ and ν is a neighbour of α in T then $G = G_\alpha *_{G_{\alpha,\nu}} G_\nu$ (cf. [8]).

We know, as the action of G on $V\Gamma$ is primitive, that G_α is a maximal subgroup of G. That implies that $G_{\alpha,\nu}$ is a maximal subgroup of G_ν; otherwise, if $G_{\alpha,\nu} < H < G_\nu$ then $G_\alpha < G_\alpha *_{G_{\alpha,\nu}} H < G$. From this it follows that

if $\nu \in VT \setminus V\Gamma$ then G_ν acts primitively on the set of vertices in $V\Gamma$ that are adjacent to ν in T.

Even just armed with these basic remarks we can start drawing conclusions about primitive graphs.

Proposition 2. *Suppose Γ is a locally finite primitive graph with infinitely many ends, and $G := \text{Aut}(\Gamma)$. Then there is a number k such that k divides the length of any G_α-orbit on $V\Gamma \setminus \{\alpha\}$.*

Proof. Let T be as above and set $k := \deg_T(\alpha)$ where $\alpha \in V\Gamma$. It is an obvious consequence of the local finiteness of Γ that all the G_α-orbits on $V\Gamma$ are finite. In particular note that since G has only two orbits on the directed edges of T we get that $k < \infty$ and that G_α acts transitively on the set of neighbours to α in T. If $\nu \in VT \setminus V\Gamma$ then we can find $\beta \in V\Gamma$ such that the geodesic in T between α and β goes through ν and the G_α-orbit of ν has to be finite because the G_α-orbit of β is finite.

Let β be a vertex in T, distinct from α, and let ν be the neighbour of α which is contained in the same component of $\Gamma \setminus \{\alpha\}$ as β. If we let l denote the size of the $G_{\alpha,\nu}$-orbit of β then, because G_α acts transitively on the neighbours of α, we get that the size of the G_α-orbit of β is kl. In particular we get that k divides the order of every G_α-orbit on $V\Gamma \setminus \{\alpha\}$. ∎

Proposition 3. *Suppose that Γ is a connected primitive graph with infinitely many ends and set $G := \text{Aut}(\Gamma)$. Then there exist distinct vertices α, β in Γ such that $\Gamma' := (V\Gamma, \{\alpha, \beta\}G)$ has connectivity one.*

Proof. Let T be as above and $\alpha \in V\Gamma$. Let now β be some vertex in $V\Gamma$ such that $d_T(\alpha, \beta) = 2$. If γ, δ are two vertices that are adjacent in Γ', then clearly $d_T(\gamma, \delta) = 2$. Suppose γ and δ are two vertices in Γ that belong to distinct components of $T \setminus \{\alpha\}$. Then $d_T(\gamma, \delta) \geq 4$. Thus it is impossible for two vertices of Γ belonging to different components of $T \setminus \{\alpha\}$ to be adjacent in Γ'. Therefore there can be no path in $\Gamma' \setminus \{\alpha\}$ between vertices that are in different components of $T \setminus \{\alpha\}$. Now we have shown that $\Gamma' \setminus \{\alpha\}$ is not connected and thus Γ' has connectivity one. ∎

Suppose that Γ is a primitive graph with $\kappa(\Gamma) = 1$. Let α be some vertex in Γ and let Λ be a block of Γ with $\alpha \in V\Lambda$. We let e_0 denote the component of $\Gamma \setminus \{\alpha\}$

that contains $\Lambda \setminus \{\alpha\}$. It is now easy to see that e_0 is a D-cut. Note that all the edges in δe_0 belong to Λ. If $\{\beta,\beta'\}$ is an edge in Γ, then some element of E must separate β and β'. Thus there is an automorphism $g \in G$ such that $\{\beta,\beta'\} \in (\delta e_0)g$. Now we see that g must map the block Λ to the block that contains $\{\beta,\beta'\}$. The conclusion is that G acts transitively on the blocks of Γ and all the blocks of Γ are isomorphic. Now it is possible to recognize $T(E)$: it is just the block-cut-vertex tree of Γ. By our remark above the stabilizer of each block acts primitively on the block.

In fact we have now just proved a part (the part we will be using) of the following theorem of Jung and Watkins, which gives a very satisfactory description of the primitive graphs with connectivity 1.

Theorem 2. ([4, Theorem 4.2]) *Let Γ be a transitive graph with $\kappa(\Gamma)=1$. Then Γ is primitive if and only if the blocks of Γ are primitive, pairwise isomorphic and have at least three vertices.*

3. Accessibility and primitivity

Theorem 3. *Every locally finite primitive graph is accessible.*

Proof. Let Γ be a locally finite primitive graph. We can clearly assume that Γ is connected and has more than one end. Set $G:=\mathrm{Aut}(\Gamma)$.

Find vertices α,β_0 such that $\Gamma_1:=(V\Gamma,\{\alpha,\beta_0\}G)$ has connectivity 1. We know that Γ_1 is quasi-isometric to Γ. Thus there is a natural bijection between the ends of Γ and Γ_1 which preserves the end structure (cf. [7, Proposition 1]). Especially, as observed in [9], Γ is accessible if and only if Γ_1 is accessible.

If L is a half-line in Γ_1 then either there is a block Λ in Γ_1 such that Λ contains all but finitely many of the vertices in L or every block in Γ_1 contains just finitely many vertices from L. In the first case we can say that the block Λ contains the end that L belongs to. Two distinct ends of Γ_1 that do not belong to the same block can clearly be separated in Γ_1 by removing just one vertex. The real problem is to separate two ends that are contained in the same block. Let Λ_1 be a block that contains α. Because all the blocks in Γ_1 are isomorphic we get that Γ_1 is accessible if and only if Λ_1 is accessible. Set $G^{(1)}:=G_{\{V\Lambda_1\}}$; that is, $G^{(1)}$ is the setwise stabilizer of $V\Lambda_1$.

Suppose that Λ_1 has more than one end. Then apply again the result of Proposition 2 and find a vertex β_2 in Λ_1 such that $\Gamma_2 := (V\Lambda_1,\{\alpha,\beta_2\}G^{(1)})$ has connectivity 1. Set $G^{(2)}:=G_{\{\Lambda_2\}}$. Let now Λ_2 denote a block of Γ_2 that contains α. We see from the above that Γ is accessible if and only if Γ_2 is accessible, and Γ_2 is accessible if and only if Λ_2 is accessible. Now we go on to find a vertex β_3 in Γ_2 and so on. If Γ is not accessible then we can continue in this way indefinitely, the inaccessibilty of Γ guarantees that in each step Λ_n will have more than one end, indeed, Λ_n will have infinitely many ends.

To see that this can not happen we look at the size s_i of the smallest orbit of $G^{(i)}_\alpha$ on $V\Gamma_i$. We know that $G^{(i)}_\alpha$ acts transitively on the blocks of $V\Gamma_i$ that contain

α. Let k_i be the number of blocks in Γ_i containing α, by transitivity $k_i \geq 2$ for all i. Then

$$s_i = k_i \cdot |\text{size of the smallest orbit of } G_\alpha^{(i)} \text{ on } \Lambda_i| = k_i \cdot s_{i+1}.$$

Thus $s_i > s_{i+1}$. If Γ is not accessible then the sequence $s_1, s_2, \ldots$ will be an infinite strictly decreasing sequence of non-negative integers, which is absurd.

In some step it must therefore happen that Γ_n has just one end and is therefore accessible and so Γ must be accessible. ∎

Theorem 4. *Let Γ be a connected locally finite primitive graph with infinitely many ends and set $G := \mathrm{Aut}(\Gamma)$. Then there exist vertices α, β in Γ such that $\Gamma' := (V\Gamma, \{\alpha,\beta\}G)$ has connectivity 1 and each block of Γ' has at most one end.*

This Theorem tells us that our graph Γ has essentially the same end structure as Γ', whose end structure is very easy to understand. First we must prove a small lemma.

Lemma 1. *Let Γ be a primitive graph with connectivity 1, and let $G := \mathrm{Aut}(\Gamma)$. Suppose α, β are distinct vertices of Γ both belonging to some block Λ of Γ. Set $\Delta := (V\Lambda, \{\alpha,\beta\}G_{\{\Lambda\}})$. Then a block of Δ is also a block of $\Gamma' := (V\Gamma, \{\alpha,\beta\}G)$.*

Proof. Let Δ' denote the subgraph of Γ' induced by $V\Lambda$. Obviously $E\Delta \subseteq E\Delta'$. Suppose $\{\gamma,\delta\} \in E\Delta'$. Then there is an element $g \in G$ such that $\{\alpha,\beta\}g = \{\gamma,\delta\}$. An automorphism that maps two vertices of a block back to the same block must stabilize the block, so $\Lambda g = \Lambda$. Therefore $g \in G_{\{\Lambda\}}$ and $\{\gamma,\delta\} \in E\Delta'$. So $E\Delta = E\Delta'$ and $\Delta = \Delta'$.

Let B be some block of Δ. Because $\Delta = \Delta'$ we know that B is also a 2-connected subgraph of Γ' and thus contained in some block B' of Γ'. We have to prove that all the vertices of B' are contained in some block of Γ, so $VB' \subseteq V\Lambda$ and therefore B' is a subgraph of Δ'. Then it follows that $B' = B$.

Let B' be a block of Γ'. Because of transitivity we can assume that B' is the block that contains the edge $\{\alpha,\beta\}$. We want to show that $VB' \subseteq V\Lambda$. If B' contains just the two vertices α and β then there is nothing to do, otherwise we know that any two edges in B' lie on a common cycle. So it is enough to show that if $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a cycle in B' with $\alpha = \alpha_1$ and $\beta = \alpha_2$ then $\alpha_1, \alpha_2, \ldots, \alpha_n$ belong to $V\Lambda$. Let T be the block-cut-vertex tree of Γ. As mentioned above, the tree T is also a structure tree of Γ and Γ'. Because α and β are in the same block of Γ we know that $d_T(\alpha,\beta) = 2$, and thus for every pair γ, δ of adjacent vertices in Γ' we know that $d_T(\gamma,\delta) = 2$. The cycle in Γ' gives us a closed path $\alpha_1, \nu_1, \alpha_2, \ldots, \nu_{n-1}, \alpha_n$ where ν_i is the vertex in T corresponding to the block in Γ that contains the edge $\{\alpha_i, \alpha_{i+1}\}$. Because all the vertices $\alpha_1, \alpha_2, \ldots, \alpha_n$ are distinct we see that $\nu_1 = \nu_2 = \cdots = \nu_{n-1}$ and ν_1 corresponds to Λ. So all the vertices $\alpha_1, \alpha_2, \ldots, \alpha_n$ are contained in $V\Lambda$ and therefore $VB' \subseteq V\Lambda$. ∎

Proof of Theorem 4. Let us continue with the notation from the proof of Theorem 3. Suppose that the blocks of Γ_n contain at most one end each. Set $\beta := \beta_n$.

If $n = 1$ then there is nothing to prove. Otherwise we apply Lemma 1 to Γ_1 to find that Λ_2 is a block of $(V\Gamma, \{\alpha,\beta_2\}G)$ and so on. In this way we find that Λ_n is a block of Γ'. ∎

Corollary 1. *Let* Γ *be a locally finite primitive graph. If* Γ *has any thick ends then* $\mathrm{Aut}(\Gamma)$ *acts transitively on the set of thick ends of* Γ.

Proof. Apply Theorem 4 above to find Γ'. Each thick ends lives inside a block of Γ' and $\mathrm{Aut}(\Gamma)$ acts transitively on the blocks of Γ'. ∎

Remark. Theorem 2 can be rephrased to say that any inaccessible graph is imprimitive. Similarly the result of Jung and Watkins from [5] saying that no infinite locally finite primitive graph has connectivity 2 can be rephrased as saying that every locally finite graph with connectivity 2 is imprimitive. In both cases one could hope that if we could find the blocks of imprimitivity then we would have found a good way to describe these graphs.

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